

NEW ESTIMATES ON GENERALIZATION OF SOME INTEGRAL INEQUALITIES FOR QUASI-CONVEX FUNCTIONS AND THEIR APPLICATIONS

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ABSTRACT. In this paper, we derive new estimates for the remainder term of the midpoint, trapezoid, and Simpson formulae for functions whose derivatives in absolute value at certain power are quasi-convex. Some applications to special means of real numbers are also given.

1. INTRODUCTION

Let $f : I \subset \mathbb{R} \rightarrow \mathbb{R}$ be a convex function defined on the interval I of real numbers and $a, b \in I$ with $a < b$. The following inequality

$$(1.1) \quad f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}.$$

holds. This double inequality is known in the literature as Hermite-Hadamard integral inequality for convex functions. Note that some of the classical inequalities for means can be derived from (1.1) for appropriate particular selections of the mapping f . Both inequalities hold in the reversed direction if f is concave. See [1, 3, 4, 6, 7, 9], the results of the generalization, improvement and extension of the famous integral inequality (1.1).

The notion of quasi-convex functions generalizes the notion of convex functions. More precisely, a function $f : [a, b] \rightarrow \mathbb{R}$ is said quasi-convex on $[a, b]$ if

$$f(\alpha x + (1-\alpha)y) \leq \sup\{f(x), f(y)\},$$

for any $x, y \in [a, b]$ and $\alpha \in [0, 1]$. Clearly, any convex function is a quasi-convex function. Furthermore, there exist quasi-convex functions which are not convex (see [7]).

The following inequality is well known in the literature as Simpson's inequality.

Let $f : [a, b] \rightarrow \mathbb{R}$ be a four times continuously differentiable mapping on (a, b) and $\|f^{(4)}\|_{\infty} = \sup_{x \in (a, b)} |f^{(4)}(x)| < \infty$. Then the following inequality holds:

$$\left| \frac{1}{3} \left[\frac{f(a)+f(b)}{2} + 2f\left(\frac{a+b}{2}\right) \right] - \frac{1}{b-a} \int_a^b f(x)dx \right| \leq \frac{1}{2880} \|f^{(4)}\|_{\infty} (b-a)^2.$$

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In recent years many authors have studied error estimations for Simpson's inequality; for refinements, counterparts, generalizations and new Simpson's type inequalities, see [2, 5, 10, 11, 12]

In [3], Alomari et al. established some upper bound for the right -hand side of Hadamard's inequality for quasi-convex mappings, The authors obtained the following results:

Theorem 1. *Let $f : I \subset \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L[a, b]$, where $a, b \in I$ with $a < b$. If $|f'|^{p/(p-1)}$ is an quasi-convex on $[a, b]$, for $p > 1$, then the following inequality holds:*

$$(1.2) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{4(p+1)^{1/p}} \left[\left(\sup \left\{ \left| f' \left(\frac{a+b}{2} \right) \right|^{\frac{p}{p-1}}, |f'(b)|^{\frac{p}{p-1}} \right\} \right)^{\frac{p-1}{p}} + \left(\sup \left\{ \left| f' \left(\frac{a+b}{2} \right) \right|^{\frac{p}{p-1}}, |f'(a)|^{\frac{p}{p-1}} \right\} \right)^{\frac{p-1}{p}} \right].$$

Theorem 2. *Let $f : I^\circ \subset \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I° , $a, b \in I^\circ$ with $a < b$. If $|f'|^q$ is an quasi-convex on $[a, b]$, for $q \geq 1$, then the following inequality holds:*

$$(1.3) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{8} \left[\left(\sup \left\{ \left| f' \left(\frac{a+b}{2} \right) \right|^q, |f'(b)|^q \right\} \right)^{\frac{1}{q}} + \left(\sup \left\{ \left| f' \left(\frac{a+b}{2} \right) \right|^q, |f'(a)|^q \right\} \right)^{\frac{1}{q}} \right].$$

In this paper, in order to provide a unified approach to establish midpoint inequality, trapezoid inequality and Simpson's inequality for functions whose derivatives in absolute value at certain power are quasi-convex, we need the following lemma given by Iscan in [8]:

Lemma 1. *Let $f : I \subset \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L[a, b]$, where $a, b \in I$ with $a < b$ and $\theta, \lambda \in [0, 1]$, then the following equality holds:*

$$(1.4) \quad \begin{aligned} & (1-\theta)(\lambda f(a) + (1-\lambda)f(b)) + \theta f((1-\lambda)a + \lambda b) - \frac{1}{b-a} \int_a^b f(x) dx \\ &= (b-a) \left[-\lambda^2 \int_0^1 (t-\theta) f'(ta + (1-t)[(1-\lambda)a + \lambda b]) dt \right. \\ & \quad \left. + (1-\lambda)^2 \int_0^1 (t-\theta) f'(tb + (1-t)[(1-\lambda)a + \lambda b]) dt \right]. \end{aligned}$$

2. MAIN RESULTS

Theorem 3. *Let $f : I \subset \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L[a, b]$, where $a, b \in I^\circ$ with $a < b$ and $\alpha, \lambda \in [0, 1]$. If $|f'|^q$ is quasi-convex on*

$[a, b]$, $q \geq 1$, then the following inequality holds:

$$\begin{aligned}
 & \left| (1-\theta)(\lambda f(a) + (1-\lambda)f(b)) + \theta f((1-\lambda)a + \lambda b) - \frac{1}{b-a} \int_a^b f(x) dx \right| \\
 & \leq (b-a) \left(\theta^2 - \theta + \frac{1}{2} \right) \left[\lambda^2 (\sup \{|f'(a)|^q, |f'(C)|^q\})^{\frac{1}{q}} \right. \\
 (2.1) \quad & \left. + (1-\lambda)^2 (\sup \{|f'(b)|^q, |f'(C)|^q\})^{\frac{1}{q}} \right]
 \end{aligned}$$

where $C = (1-\lambda)a + \lambda b$.

Proof. Suppose that $q \geq 1$ and $C = (1-\lambda)a + \lambda b$. From Lemma 3 and using the well known power mean inequality, we have

$$\begin{aligned}
 & \left| (1-\theta)(\lambda f(a) + (1-\lambda)f(b)) + \theta f(C) - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq (b-a) \\
 & \left[\lambda^2 \int_0^1 |t-\theta| |f'(ta + (1-t)C)| dt + (1-\lambda)^2 \int_0^1 |t-\theta| |f'(tb + (1-t)C)| dt \right] \\
 & \leq (b-a) \left\{ \lambda^2 \left(\int_0^1 |t-\theta| dt \right)^{1-\frac{1}{q}} \left(\int_0^1 |t-\theta| |f'(ta + (1-t)C)|^q dt \right)^{\frac{1}{q}} \right. \\
 (2.2) \quad & \left. + (1-\lambda)^2 \left(\int_0^1 |t-\theta| dt \right)^{1-\frac{1}{q}} \left(\int_0^1 |t-\theta| |f'(tb + (1-t)C)|^q dt \right)^{\frac{1}{q}} \right\}.
 \end{aligned}$$

Since $|f'|^q$ is quasi-convex on $[a, b]$, we know that for $t \in [0, 1]$

$$|f'(ta + (1-t)C)|^q \leq \sup \{|f'(a)|^q, |f'(C)|^q\},$$

and

$$|f'(tb + (1-t)C)|^q \leq \sup \{|f'(a)|^q, |f'(C)|^q\}.$$

Hence, by simple computation

$$(2.3) \quad \int_0^1 |t-\theta| dt = \theta^2 - \theta + \frac{1}{2},$$

$$(2.4) \quad \int_0^1 |t-\theta| |f'(ta + (1-t)C)|^q dt = \left(\theta^2 - \theta + \frac{1}{2} \right) \sup \{|f'(a)|^q, |f'(C)|^q\},$$

and

$$(2.5) \quad \int_0^1 |t-\theta| |f'(tb + (1-t)C)|^q dt = \left(\theta^2 - \theta + \frac{1}{2} \right) \sup \{|f'(b)|^q, |f'(C)|^q\}.$$

Thus, using (2.3)-(2.5) in (2.2), we obtain the inequality (2.1). This completes the proof. \square

Corollary 1. *Under the assumptions of Theorem 3 with $q = 1$, the inequality (2.1) reduced to the following inequality*

$$\begin{aligned} & \left| (1 - \theta) (\lambda f(a) + (1 - \lambda) f(b)) + \theta f((1 - \lambda)a + \lambda b) - \frac{1}{b - a} \int_a^b f(x) dx \right| \\ & \leq (b - a) \left(\theta^2 - \theta + \frac{1}{2} \right) [\lambda^2 \sup \{|f'(a)|, |f'(C)|\} \\ & \quad + (1 - \lambda)^2 \sup \{|f'(b)|, |f'(C)|\}]. \end{aligned}$$

Corollary 2. *Under the assumptions of Theorem 3 with $\lambda = \frac{1}{2}$ and $\theta = \frac{2}{3}$, from the inequality (2.1) we get the following Simpson type inequality*

$$\begin{aligned} & \left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] - \frac{1}{b-a} \int_a^b f(x) dx \right| \\ & \leq (b-a) \left(\frac{5}{72} \right) \left[\left(\sup \left\{ |f'(a)|^q, \left| f'\left(\frac{a+b}{2}\right) \right|^q \right\} \right)^{\frac{1}{q}} + \left(\sup \left\{ |f'(b)|^q, \left| f'\left(\frac{a+b}{2}\right) \right|^q \right\} \right)^{\frac{1}{q}} \right]. \end{aligned}$$

Corollary 3. *Under the assumptions of Theorem 3 with $\lambda = \frac{1}{2}$ and $\theta = 1$, from the inequality (2.1) we get the following midpoint inequality*

$$\begin{aligned} & \left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a} \int_a^b f(x) dx \right| \\ & \leq \frac{b-a}{8} \left[\left(\sup \left\{ |f'(a)|^q, \left| f'\left(\frac{a+b}{2}\right) \right|^q \right\} \right)^{\frac{1}{q}} + \left(\sup \left\{ |f'(b)|^q, \left| f'\left(\frac{a+b}{2}\right) \right|^q \right\} \right)^{\frac{1}{q}} \right]. \end{aligned}$$

Corollary 4. *Under the assumptions of Theorem 3 with $\lambda = \frac{1}{2}$ and $\theta = 0$, from the inequality (2.1) we get the following trapezoid inequality*

$$\begin{aligned} & \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \\ & \leq \frac{b-a}{8} \left[\left(\sup \left\{ |f'(a)|^q, \left| f'\left(\frac{a+b}{2}\right) \right|^q \right\} \right)^{\frac{1}{q}} + \left(\sup \left\{ |f'(b)|^q, \left| f'\left(\frac{a+b}{2}\right) \right|^q \right\} \right)^{\frac{1}{q}} \right]. \end{aligned}$$

which is the same of the inequality (1.3).

Using Lemma 1 we shall give another result for convex functions as follows.

Theorem 4. *Let $f : I \subset \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L[a, b]$, where $a, b \in I^\circ$ with $a < b$ and $\alpha, \lambda \in [0, 1]$. If $|f'|^q$ is quasi-convex on*

$[a, b]$, $q > 1$, then the following inequality holds:

$$\begin{aligned}
 (2.6) \quad & \left| (1 - \theta) (\lambda f(a) + (1 - \lambda) f(b)) + \theta f((1 - \lambda) a + \lambda b) - \frac{1}{b - a} \int_a^b f(x) dx \right| \\
 & \leq (b - a) \left(\frac{\theta^{p+1} + (1 - \theta)^{p+1}}{p + 1} \right)^{\frac{1}{p}} \left[\lambda^2 (\sup \{|f'(a)|^q, |f'(C)|^q\})^{\frac{1}{q}} \right. \\
 & \quad \left. + (1 - \lambda)^2 (\sup \{|f'(b)|^q, |f'(C)|^q\})^{\frac{1}{q}} \right]
 \end{aligned}$$

where $C = (1 - \lambda) a + \lambda b$ and $\frac{1}{p} + \frac{1}{q} = 1$.

Proof. Suppose that $C = (1 - \lambda) a + \lambda b$. From Lemma 3 and by Hölder's integral inequality, we have

$$\begin{aligned}
 & \left| (1 - \theta) (\lambda f(a) + (1 - \lambda) f(b)) + \theta f(C) - \frac{1}{b - a} \int_a^b f(x) dx \right| \leq (b - a) \\
 & \left[\lambda^2 \int_0^1 |t - \theta| |f'(ta + (1 - t)C)| dt + (1 - \lambda)^2 \int_0^1 |t - \theta| |f'(tb + (1 - t)C)| dt \right] \\
 & \leq (b - a) \left\{ \lambda^2 \left(\int_0^1 |t - \theta|^p dt \right)^{\frac{1}{p}} \left(\int_0^1 |f'(ta + (1 - t)C)|^q dt \right)^{\frac{1}{q}} \right. \\
 (2.7) \quad & \left. + (1 - \lambda)^2 \left(\int_0^1 |t - \theta|^p dt \right)^{\frac{1}{p}} \left(\int_0^1 |f'(tb + (1 - t)C)|^q dt \right)^{\frac{1}{q}} \right\}.
 \end{aligned}$$

Since $|f'|^q$ is quasi-convex on $[a, b]$, we get

$$(2.8) \quad \int_0^1 |f'(ta + (1 - t)C)|^q dt = \sup \{|f'(a)|^q, |f'(C)|^q\}$$

Similarly,

$$(2.9) \quad \int_0^1 |f'(tb + (1 - t)C)|^q dt = \sup \{|f'(b)|^q, |f'(C)|^q\}.$$

By simple computation

$$(2.10) \quad \int_0^1 |t - \theta|^p dt = \frac{\theta^{p+1} + (1 - \theta)^{p+1}}{p + 1},$$

thus, using (2.8)-(2.10) in (2.7), we obtain the inequality (2.6). This completes the proof. \square

Corollary 5. *Under the assumptions of Theorem 4 with $\lambda = \frac{1}{2}$ and $\theta = \frac{2}{3}$, from the inequality (2.6) we get the following Simpson type inequality*

$$\begin{aligned} & \left| \frac{1}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] - \frac{1}{b-a} \int_a^b f(x) dx \right| \\ & \leq \frac{b-a}{12} \left(\frac{1+2^{p+1}}{3(p+1)} \right)^{\frac{1}{p}} \left\{ \left(\sup \left\{ \left| f'\left(\frac{a+b}{2}\right) \right|^q, |f'(a)|^q \right\} \right)^{\frac{1}{q}} \right. \\ & \quad \left. + \left(\sup \left\{ \left| f'\left(\frac{a+b}{2}\right) \right|^q, |f'(b)|^q \right\} \right)^{\frac{1}{q}} \right\}. \end{aligned}$$

Corollary 6. *Under the assumptions of Theorem 4 with $\lambda = \frac{1}{2}$ and $\theta = 0$, from the inequality (2.6) we get the following trapezoid inequality*

$$\begin{aligned} \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| & \leq \frac{b-a}{4(p+1)^{1/p}} \left[\left\{ \left(\sup \left\{ \left| f'\left(\frac{a+b}{2}\right) \right|^q, |f'(a)|^q \right\} \right)^{\frac{1}{q}} \right. \right. \\ & \quad \left. \left. + \left(\sup \left\{ \left| f'\left(\frac{a+b}{2}\right) \right|^q, |f'(b)|^q \right\} \right)^{\frac{1}{q}} \right\} \right]. \end{aligned}$$

which is the same of the inequality (1.2).

Corollary 7. *Under the assumptions of Theorem 4 with $\lambda = \frac{1}{2}$ and $\theta = 1$, from the inequality (2.6) we get the following midpoint inequality*

$$\begin{aligned} \left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a} \int_a^b f(x) dx \right| & \leq \frac{b-a}{4(p+1)^{1/p}} \left[\left\{ \left(\sup \left\{ \left| f'\left(\frac{a+b}{2}\right) \right|^q, |f'(a)|^q \right\} \right)^{\frac{1}{q}} \right. \right. \\ & \quad \left. \left. + \left(\sup \left\{ \left| f'\left(\frac{a+b}{2}\right) \right|^q, |f'(b)|^q \right\} \right)^{\frac{1}{q}} \right\} \right], \end{aligned}$$

which is the better than the inequality in [1, Corollary 8].

3. SOME APPLICATIONS FOR SPECIAL MEANS

Let us recall the following special means of arbitrary real numbers a, b with $a \neq b$ and $\alpha \in [0, 1]$:

- (1) The weighted arithmetic mean

$$A_\alpha(a, b) := \alpha a + (1 - \alpha)b, \quad a, b \in \mathbb{R}.$$

- (2) The unweighted arithmetic mean

$$A(a, b) := \frac{a+b}{2}, \quad a, b \in \mathbb{R}.$$

- (3) The weighted harmonic mean

$$H_\alpha(a, b) := \left(\frac{\alpha}{a} + \frac{1-\alpha}{b} \right)^{-1}, \quad a, b \in \mathbb{R} \setminus \{0\}.$$

(4) The unweighted harmonic mean

$$H(a, b) := \frac{2ab}{a+b}, \quad a, b \in \mathbb{R} \setminus \{0\}.$$

(5) The Logarithmic mean

$$L(a, b) := \frac{b-a}{\ln b - \ln a}, \quad a, b > 0, \quad a \neq b.$$

(6) Then n-Logarithmic mean

$$L_n(a, b) := \left(\frac{b^{n+1} - a^{n+1}}{(n+1)(b-a)} \right)^{\frac{1}{n}}, \quad n \in \mathbb{N}, \quad a, b \in \mathbb{R}, \quad a \neq b.$$

Proposition 1. *Let $a, b \in \mathbb{R}$ with $a < b$, and $n \in \mathbb{N}$, $n \geq 2$. Then, for $\theta, \lambda \in [0, 1]$ and $q \geq 1$, we have the following inequality:*

$$\begin{aligned} & |(1-\theta)A_\lambda(a^n, b^n) + \theta A_\lambda^n(a, b) - L_n^n(a, b)| \\ & \leq (b-a) \left(\theta^2 - \theta + \frac{1}{2} \right) n \left[\lambda^2 \left(\sup \left\{ |a|^{(n-1)q}, |A_\lambda(b, a)|^{(n-1)q} \right\} \right)^{\frac{1}{q}} \right. \\ & \quad \left. + (1-\lambda)^2 \left(\sup \left\{ |b|^{(n-1)q}, |A_\lambda(b, a)|^{(n-1)q} \right\} \right)^{\frac{1}{q}} \right]. \end{aligned}$$

Proof. The assertion follows from Theorem 3, for $f(x) = x^n$, $x \in \mathbb{R}$. \square

Proposition 2. *Let $a, b \in \mathbb{R}$ with $a < b$, and $n \in \mathbb{N}$, $n \geq 2$. Then, for $\theta, \lambda \in [0, 1]$ and $q > 1$, we have the following inequality:*

$$\begin{aligned} & |(1-\theta)A_\lambda(a^n, b^n) + \theta A_\lambda^n(a, b) - L_n^n(a, b)| \\ & \leq (b-a) \left(\frac{\theta^{p+1} + (1-\theta)^{p+1}}{p+1} \right)^{\frac{1}{p}} n \left[\lambda^2 \left(\sup \left\{ |a|^{(n-1)q}, |A_\lambda(b, a)|^q \right\} \right)^{\frac{1}{q}} \right. \\ & \quad \left. + (1-\lambda)^2 \left(\sup \left\{ |b|^{(n-1)q}, |A_\lambda(b, a)|^q \right\} \right)^{\frac{1}{q}} \right] \end{aligned}$$

where $\frac{1}{p} + \frac{1}{q} = 1$.

Proof. The assertion follows from Theorem 4, for $f(x) = x^n$, $x \in \mathbb{R}$. \square

Proposition 3. *Let $a, b \in \mathbb{R}$ with $0 < a < b$, and $\theta, \lambda \in [0, 1]$. Then, for $q \geq 1$, we have the following inequality:*

$$\begin{aligned} & |(1-\theta)H_\lambda^{-1}(a, b) + \theta A_\lambda^{-1}(a, b) - L^{-1}(a, b)| \\ & \leq (b-a) \left(\theta^2 - \theta + \frac{1}{2} \right) \left[\lambda^2 \left(\sup \left\{ a^{-2q}, A_\lambda(b, a)^{-2q} \right\} \right)^{\frac{1}{q}} \right. \\ & \quad \left. + (1-\lambda)^2 \left(\sup \left\{ b^{-2q}, A_\lambda(b, a)^{-2q} \right\} \right)^{\frac{1}{q}} \right]. \end{aligned}$$

Proof. The assertion follows from Theorem 3., for $f(x) = \frac{1}{x}$, $x \in (0, \infty)$. \square

Proposition 4. *Let $a, b \in \mathbb{R}$ with $0 < a < b$, and $\theta, \lambda \in [0, 1]$. Then, for $q > 1$, we have the following inequality:*

$$\begin{aligned} & |(1 - \theta) H_{\lambda}^{-1}(a, b) + \theta A_{\lambda}^{-1}(a, b) - L^{-1}(a, b)| \\ & \leq (b - a) \left(\frac{\theta^{p+1} + (1 - \theta)^{p+1}}{p + 1} \right)^{\frac{1}{p}} \left[\lambda^2 \left(\sup \left\{ a^{-2q}, A_{\lambda}(b, a)^{-2q} \right\} \right)^{\frac{1}{q}} \right. \\ & \quad \left. + (1 - \lambda)^2 \left(\sup \left\{ b^{-2q}, A_{\lambda}(b, a)^{-2q} \right\} \right)^{\frac{1}{q}} \right] \end{aligned}$$

Proof. The assertion follows from Theorem 4, for $f(x) = \frac{1}{x}$, $x \in (0, \infty)$. □

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